8.2/34

$$\int_0^{1/2} \sin^{-1} x \, dx$$

memorized rule

u-substitution

partial fractions

integration by parts

$$\int u\,dv = uv - \int v\,du$$

$u = \sin^{-1} x \qquad dv = 1\,dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \qquad v = x$

$$\left[x \sin^{-1}(x)\right]_0^{1/2} - \int_{x=0}^{x=1/2} \frac{x}{\sqrt{1-x^2}}\,dx$$

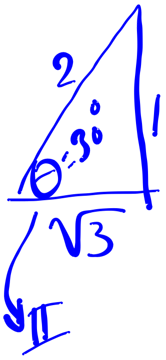
$u = 1 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$

$$\int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int_1^{3/4} u^{-1/2}\,du = -\frac{1}{2}\cdot 2u^{1/2}\Big|_1^{3/4}$$

$$-\sqrt{u}\,\Big|_1^{3/4}$$



$$\left(\frac{1}{2}\right)\sin^{-1}\left(\frac{1}{2}\right) - 0 - \left[-\sqrt{\frac{3}{4}} - \left(-\sqrt{1}\right)\right]$$

$$\frac{1}{2}\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2} - 1 = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

8.8/5

$$\int_4^\infty \frac{2}{x^2-1}\,dx =$$

memorized rule

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partial fractions

integration by parts

$\int u\,dv = uv - \int v\,du$

$$\lim_{\text{Spencer}\to\infty} \int_4^{\text{SPENCER}} \frac{2}{x^2-1}\,dx$$

$$\lim_{\text{SPENCER}\to\infty} \int_4^{\text{SPENCER}} \frac{2}{(x-1)(x+1)}\,dx$$

$$\frac{2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = A(x+1)+B(x-1)$$

$$Ax+A+Bx-B$$

$$(A+B)x+(A-B)$$

$A+B=0$ $A-B=2$

$A=-B$ $-B-B=2$

$A-1=0$ $B=-1$

$A=1$

$$\int \frac{1}{x-1}\,dx - \int \frac{1}{x+1}\,dx$$

$$\ln|x-1| - \ln|x+1| + C$$

$$\lim_{\text{SPENCER}\to\infty} \Big[\ln(x-1)-\ln(x+1)\Big]_{x=4}^{\text{SPENCER}} = \lim_{\text{SPENCER}\to\infty} \Big\{\ln(\text{SPENCER}-1) - \ln(\text{SPENCER}+1) - [\ln 3 - \ln 5]\Big\}$$

$$\lim_{b\to\infty} \ln(b-1) - \ln(b+1) - \ln 3 + \ln 5$$

$$= \lim_{b\to\infty} \ln\left(\frac{b-1}{b+1}\right) - \ln 3 + \ln 5$$

$\ln 1 = 0$

$$= -\ln 3 + \ln 5 = \ln\frac{5}{3}$$

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8.8/7)

$$\int_e^{\infty} \frac{1}{x(\ln x)^3}dx =$$

$$\ln^3 x = (\ln x)^3$$

$$\lim_{b\to\infty} \int_e^{b} \frac{1}{x(\ln x)^3}dx$$

$u = \ln x$

$du = \left(\frac{1}{x}\right)dx$

$$= \lim_{b\to\infty} \int_1^{\ln b} \frac{1}{u^3}\,du$$

$$\int u^{-3}du = \frac{u^{-2}}{-2}$$

$$= \lim_{b\to\infty} \left(-\frac{1}{2u^2}\right)\Big|_1^{\ln b} = \lim_{b\to\infty} -\frac{1}{2(\ln b)^2} - \left(-\frac{1}{2}\right)$$

$\ln b$

$$= +\frac{1}{2}$$

succinct

marked by compact precise expression without wasted words

syn: see CONCISE ant: discursive

8.8/10) $\int_{-\infty}^{2} \frac{1}{x^2+4}\,dx$

$= \lim_{a\to-\infty} \int_a^2 \frac{1}{x^2+4}\,dx$

$= \lim_{a\to-\infty} \int_a^2 \frac{1}{4\left(\frac{x^2}{4}+1\right)}\,dx = \lim_{a\to-\infty} \frac{1}{4}\int_a^2 \frac{1}{\frac{x^2}{4}+1}\,dx$

memorized rule

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$\int u\,dv = uv - \int v\,du$

$\frac{1}{u^2+1}$

$u^2 = \frac{x^2}{4} \Rightarrow u = \frac{x}{2}$

Let $u = \frac{x}{2}$

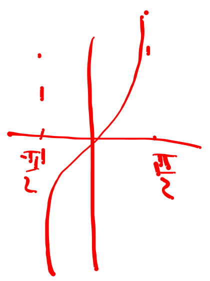
$du = \frac{1}{2}\,dx$

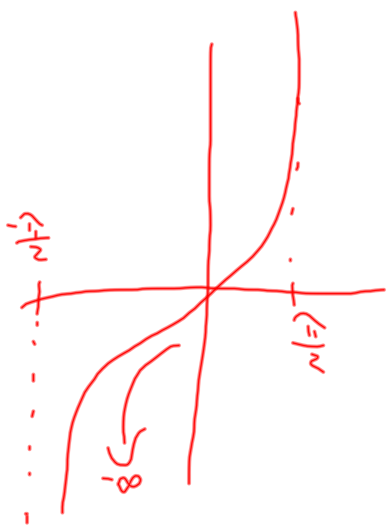
$2\,du = dx$

$= \lim_{a\to-\infty} \frac{1}{4}\int_{x=a,\ u=\frac{a}{2}}^{x=2,\ u=1} 2\left(\frac{1}{u^2+1}\right)du = \lim_{a\to-\infty} \frac{1}{2}\left(\tan^{-1} u\right)\Big|_{\frac{a}{2}}^{1}$

$= \lim_{a\to-\infty} \frac{1}{2}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{a}{2}\right)\right)$

$= \lim_{a\to-\infty} \frac{\pi}{8} - \left(-\frac{\pi}{4}\right) = \frac{3\pi}{8}$





8.2/33)

$$\int_{x=-2}^{x=2} \ln(x+3)\,dx$$

$u = x+3$

$du = dx$

$x=-2 \Rightarrow u=-2+3=1$

$x=+2 \Rightarrow u=2+3=5$

$$\int_{u=1}^{u=5} \ln u\,du = \int_1^5 \ln x\,dx$$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$

$$= x\ln x\Big|_1^5 - \int_1^5 x\left(\frac{1}{x}dx\right) = x\ln x\Big|_1^5 - \int_1^5 1\,dx$$

$$= 5\ln 5 - 1\ln 1 - \left[x\Big|_1^5\right] = 5\ln 5 - 4$$

memorized rule

u-substitution

partial fractions

integration by parts

$\int u\,dv = uv - \int v\,du$